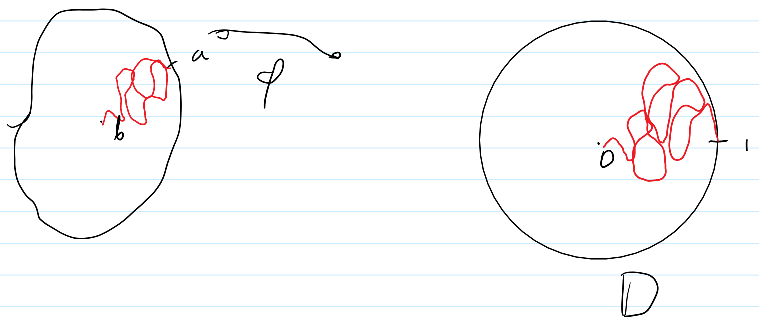
Another model: curve from some $a \in \partial\Omega$ to $b \in \Omega$.



By conformal invariance: only need to consider $a = 1, b = 0$.

Def Radial SLE_κ in $(\mathbb{D}, 1, 0)$ is the random curve in $\mathbb{D}$, such that for the conformal map g_t: component of 0 of $\mathbb{D} \setminus \gamma[0,t]$ (Ω_t) $\to \mathbb{D}$ with $g_t'(0) = e^t$, the Löwner equation

$$\frac{\partial g_t}{\partial t} = g_t(z) \frac{e^{iB(\kappa t)} + g_t(z)}{e^{iB(\kappa t)} - g_t(z)}, \quad g_0(z) = z. \quad (B(t) \text{ - standard 1D BM})$$

(For $f_t = g_t^{-1}$, we have

$$\partial f_t(z) = -z f_t'(z) \frac{e^{iB(\kappa t)} + z}{e^{iB(\kappa t)} - z} \quad)$$

The radial SLE_κ in s.c. Ω from $a \in \partial\Omega$ to $b \in \Omega$ is defined as the image of SLE_κ in $(\mathbb{D}, 1, 0)$ under conformal map $\varphi : (\mathbb{D}, 1, 0) \to (\Omega, a, b)$.

As in chordal case, satisfies conformal invariance + domain Markov property.

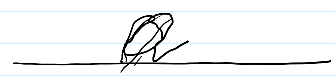
Relation between chordal and radial SLE:

Theorem. Let $a, b \in \partial\Omega$, $c \in \Omega$ (Ω - s.c. domain).

Let γ_t be radial SLE_κ from a to c.

$K_t := \Omega \setminus \Omega_t$ - hull of (γ_t)

$T := \inf\{t \geq 0 : b \in K_t\}$.



Let $\tilde{\gamma}_t$ be chordal SLE_κ from a to b,

$\tilde{K}_t$ - its hull,

$\tilde{T} := \inf\{t \geq 0 : c \in \tilde{K}_t\}$.

Then $\exists\, T_n, \tilde{T}_n$ - increasing sequences of stopping times, $T_n \uparrow T$, $\tilde{T}_n \uparrow \tilde{T}$ such that $\forall n \geq 1$,

$(\gamma_t, t \in [0, T_n])$ and $(\tilde{\gamma}_t, t \in [0, \tilde{T}_n])$ have absolutely continuous law, up to a time change.

Proof. (for $\kappa = 6$, the law is the same).

Enough to do it when $\Omega = \mathbb{D}$, $a = e^{i\theta}$, $b = 1$, $c = 0$ (by conformal invariance).

Let $\psi(z) := i\,\dfrac{1+z}{1-z}$, $\psi(\mathbb{D}) = \mathbb{H}$, $\psi(0) = i$, $\psi(1) = \infty$

Let $\tilde{B}_u$ - 1D Brownian motion with $\tilde{B}_0 = \psi(e^{i\theta})$

Chordal SLE_κ in $\mathbb{D}$ from $e^{i\theta}$ to 1 is

$$\frac{\partial \tilde{g}_u}{\partial u} = \frac{2}{\tilde{g}_u - \tilde{B}(\kappa u)}, \quad \tilde{g}_0 = \psi(z). \qquad \tilde{g}_u : \mathbb{D} \setminus K_u \to \mathbb{H}.$$

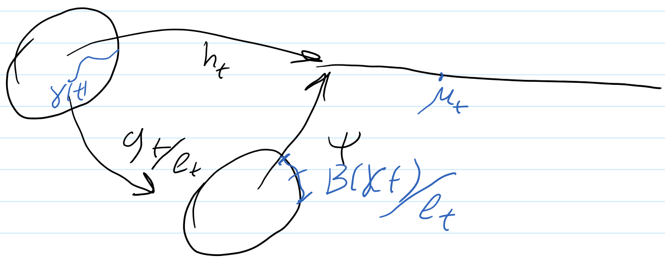
$\tilde{T}_z := \{\min u : \tilde{g}_u(z) = \tilde{B}(\kappa u)\}$.

Radial SLE_κ in $\mathbb{D}$ from $e^{i\theta}$ to 0:

$$\frac{\partial g_t}{\partial t} = g_t \, \frac{e^{iB(\kappa t)} + g_t(z)}{\ldots} \qquad g_0(z) = e^{i\theta} \ldots$$

$$\frac{\partial g_t}{\partial t} = g_t \frac{e^{iB(\kappa t)} + g_t(z)}{e^{iB(\kappa t)} - g_t(z)} \qquad g_0(z) = e^{i\theta} z.$$

Let $e_t := g_t(1)$, $\quad h_t(z) := \psi\left(\frac{g_t(z)}{e_t}\right)$, $\quad \mu_t := \psi\left(\frac{e^{iB(\kappa t)}}{e_t}\right)$.



Then

$$\frac{\partial h_t}{\partial t} = \frac{(1+\mu_t^2)(1+h_t^2)}{2(h_t - \mu_t)}$$

Define linear transformation $\varphi_t(z) = a(t)z + b(t)$ by

$a(0) = 1, \quad \partial_t a = (1+\mu_t^2)\frac{a}{2}$

$b(0) = 0, \quad \partial_t b = -(1+\mu_t^2)\frac{a\mu_t}{2}.$

Let $m_t(z) := \varphi_t \circ h_t(z)$, $\quad \beta_t := \varphi_t(\mu_t)$

Then

$$\partial_t m_t = \frac{(1+\mu_t^2)^2 \frac{a^2}{2}}{(m_t - \beta_t)}.$$

Let us change time to get rid of the factors: $\partial_t u = \frac{(1+\mu_t^2)a^2}{4}$.

Then

$$\frac{\partial m}{\partial u} = \frac{2}{m_u - \beta_u}.$$ Chordal!

What is β_u?

By Ito:

$$d\mu_t = \frac{(1+\mu_t^2)}{2}\sqrt{\kappa}\, dB_t + \frac{\gamma_t(1+\gamma_t^2)}{2}\left(\frac{\kappa}{2} - 1\right)dt$$

so

$$d\beta_t = (1+\gamma_t^2)\, a(t)\, (\sqrt{\kappa}\, dB \ldots$$

So

$$d\beta_t = \frac{(1+\gamma_t^2)\,a(t)}{2}\left(\sqrt{\kappa}\, dB_t + \left(\frac{\kappa}{2} - 3\right)\gamma_t\, dt\right)$$

$\kappa = 6 \implies \frac{\kappa}{2} - 3 = 0 \implies$

$\boxed{d\beta_u = \sqrt{6}\, dB_u}$ – driven by $B(6u)$